Received: *July 26, 2000*, Accepted: *September 22, 2000*
Revised: *August 19, 2000*

LR scalar mixings and 1-loop neutrino masses

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Abstract: Within the framework of the complete theory of supersymmetry without R-parity, where all possible R-parity violating terms are admitted, we perform a systematic analytical study of all sources of neutrino masses up to "direct 1-loop" (defined explicitly below) level. In the passing, we present the full result for squark and slepton masses. In particular, there are interesting LR squark and slepton mixings, which involve both bilinear and trilinear R-parity violating parameters. The existence and important phenomenological implications of such terms have been largely overlooked in previous studies. In particular, in the studies under which either one type of the couplings is assumed to vanish or neglected, the terms would not show up. The LR mixings play a central role in neutrino mass generation. Our results look straight forward to be obtained, which, in our opinion, is an illustration of the effectiveness of our formulation adopted.

Keywords: Solar and Atmospheric Neutrinos, Supersymmetric Standard Model, Neutrino Physics.

Contents

1. Introduction

The minimal supersymmetric standard model (MSSM) is no doubt the most popular candidate theory for physics beyond the Standard Model (SM). The alternative theory with a discrete symmetry called R-parity not imposed deserves no less attention. In particular, the latter admits neutrino masses, without the need for any extra matter field beyond the minimal spectrum. At the present time, experimental results from neutrino physics [1] is actually the only data we have demanding physics beyond the SM, while signals from supersymmetry (SUSY) are still absent. The neutrino data provides strong hints for the existence of Majorana type masses. The latter means lepton number violation, which is suggestive of R-parity violation. Hence, it is easy to appreciate the interest in R-parity violating (RPV) contributions to neutrino masses. The study of this topic has a long history, starting from ref. [2]. Two of the notable papers on different aspects of the topic are given in refs. [3, 4], to which readers are also referred for references to earlier works. More recent works in the subject area [5]–[10] mainly focus on the fitting of the neutrino oscillation data under different scenarios while a comprehensive analysis of all the RPV contributions is still missing. This paper aims at providing such a picture.

JHEP09(2000)037

Like most of the other recent studies, we will focus on the sub-eV neutrino mass scale suggested by the Super-Kamiokande atmospheric neutrino data [11], though most of our results are actually valid for a much larger range of neutrino masses. As illustrated below, there is a tree-level but seesaw suppressed contribution and some direct 1-loop contributions. Our level of treatment in this paper stops there, i.e. we will, in general, not go into contributions that are expected to be further suppressed. There are direct 1-loop contributions which involve a further seesaw type suppression hidden inside the loop. These are 1-loop diagrams that would suggest a null result if electroweak (EW) states are used for the particles running inside the loop and only a minimal number of mass insertions is admitted. When one thinks about the exact result to be obtained from using mass eigenstates instead, a non-zero result could emerge. It is diffcult to give analytical expressions for the mass eigenstate results. An approximation to the latter could be obtained by considering the EW state diagrams with extra mass insertions. In the case that these mass insertions are RPV, it typically means extra seesaw type suppression. We refer to contributions from such diagrams as pseudo-direct 1-loop contributions, which we will discuss without giving explicit formulae. We list also the well-known results. The idea here is to perform a systematic analysis and present the exhaustive list of all contributions up to the level of treatment.

A similar comprehensive listing of neutrino mass contributions up to the 1-loop level (direct or indirect) has been presented in ref. [12]. However, the latter analysis is limited to a scenario where the "third generation couplings dominate". This amounts to admitting only non-zero λ'_{i33}'s and λ_{i33}'s among the trilinear RPV couplings, though all non-zero bilinear RPV are indeed included by the authors. In our opinion, the maximal mixing result from Super-Kamiokande [11] brings the wisdom of "third generation domination" under question. Refs [7, 9], for example illustrate how no (family) hierarchy, or even an anti-hierarchy, among the RPV couplings may be preferred. The present analysis handles the complete theory of supersymmetry (SUSY) without R-parity, where all kind of RPV terms are admitted without bias. We present complete tree-level mass matrices for the scalars in this generic scenario. There is another major difference between the two studies. Ref. [12] is interested in performing some numerical calculation. While the latter is important for explicit fitting of experimental numbers, much of the physical origin of the neutrino mass contributions are hidden under elements of mixing matrices parametrizing the effective couplings of the neutrinos to squark or slepton mass eigenstates. We are interested here in illustrating the explicit origin of each contribution. Hence, we stay with electroweak (EW) state notation and give diagrammatic as well as analytical expressions of each individual contribution. Of particular interest here is a new type of contribution involving a RPV LR scalar (squark or slepton) mixings, which has been larger overlooked by previous authors.

We hope that results here will be useful for a better understanding the role of each RPV parameter and identifying interesting regions of the extensive parameter space.

To study all the RPV contributions in a single consistent framework, one needs an effective formulation of the complete theory of SUSY without R-parity. The latter theory is generally better motivated than *ad hoc* versions of RPV theories. The large number of new parameters involved, however, makes the theory difficult to analyze. It has been illustrated [13] that an optimal parametrization, called the single-VEV parametrization, can be of great help in making the task manageable. The effectiveness of the SVP has been explored to perform an extensive study on the resultant leptonic phenomenology [13], to identify new type of neutrino mass contributions [9], and to study a new contribution to neutron electric dipole moment at 1-loop level [14, 15], as well as new sources of contribution to flavor changing neutral current processes such as $b \to s\,\gamma$ [16] and $\mu \to e\,\gamma$ [17]. Studies of neutrino masses and mixings under the formulation also include refs. [7, 8]. In fact, neutrino masses contribution is a central aspect of RPV effects and is likely to provide the most stringent bounds on the couplings, though many of the bounds obtained depend on assumptions on interpretation of neutrino data and could be relaxed or removed by simple extensions of the theory allowing extra sterile neutrino(s).

One-loop neutrino mass generation in SUSY without R-parity typically involves LR mixings of squarks or slepton. We want to emphasize again that squark and slepton mass matrices presented here are complete, with all source of R-parity violation included. Such results are explicitly presented for the first time. We consider the results to be interesting in their own right.

In the appendix, we give also an explicit illustration that all the VEV's under the SVP may be taken as real, despite the existence of complex parameters in the scalar potential; and give some important consistence relationships among some of the parameters involved. These results have not been published before, and serve as important background for clarifying some issues on the scalar masses and neutrino mass contributions discussed.

2. Formulation and notation

We summarize our formulation and notation below. The most general renormalizable superpotential for the supersymmetric SM (without R-parity) can be written as

$$W = \varepsilon_{ab}\left[\mu_\alpha \hat{H}_u^a \hat{L}_\alpha^b + h_{ik}^u \hat{Q}_i^a \hat{H}_u^b \hat{U}_k^C + \lambda'_{\alpha jk} \hat{L}_\alpha^a \hat{Q}_j^b \hat{D}_k^C + \frac{1}{2}\,\lambda_{\alpha\beta k} \hat{L}_\alpha^a \hat{L}_\beta^b \hat{E}_k^C\right] + \\ + \frac{1}{2}\,\lambda''_{ijk} \hat{U}_i^C \hat{D}_j^C \hat{D}_k^C\,, \tag{2.1}$$

where (a, b) are SU(2) indices, (i, j, k) are the usual family (flavor) indices, and (α, β) are extended flavor index going from 0 to 3. In the limit where $\lambda_{ijk}, \lambda'_{ijk}, \lambda''_{ijk}$ and

μ_i all vanish, one recovers the expression for the R-parity preserving case, with $\hat{L}_0$ identified as $\hat{H}_d$. Without R-parity imposed, the latter is not *a priori* distinguishable from the $\hat{L}_i$'s. Note that λ is antisymmetric in the first two indices, as required by the SU(2) product rules, as shown explicitly here with $\varepsilon_{12} = -\varepsilon_{21} = 1$. Similarly, λ'' is antisymmetric in the last two indices from $\mathrm{SU}(3)_C$.

R-parity is exactly an *ad hoc* symmetry put in to make $\hat{L}_0$, stand out from the other $\hat{L}_i$'s as the candidate for $\hat{H}_d$. It is defined in terms of baryon number, lepton number, and spin as, explicitly, $\mathcal{R} = (-1)^{3B+L+2S}$. The consequence is that the accidental symmetries of baryon number and lepton number in the SM are preserved, at the expense of making particles and superparticles having a categorically different quantum number, R-parity. The latter is actually not the most effective discrete symmetry to control superparticle mediated proton decay [18], but is most restrictive in terms of what is admitted in the lagrangian, or the superpotential alone.

A naive look at the scenario suggests that the large number of new couplings makes the task formidable. However, it becomes quite manageable with an optimal choice of flavor bases, the SVP [13]. In fact, doing phenomenological studies without specifying a choice of flavor bases is ambiguous. It is like doing SM quark physics with 18 complex Yukawa couplings instead of the 10 real physical parameters. In SUSY without R-parity, the choice of an optimal parametrization mainly concerns the 4 $\hat{L}_\alpha$ flavors. Under the SVP, flavor bases are chosen such that : 1/ among the $\hat{L}_\alpha$'s, only $\hat{L}_0$, bears a VEV, i.e. $\langle \hat{L}_i \rangle \equiv 0$; 2/ $h^e_{jk} (\equiv \lambda_{0jk} = -\lambda_{j0k}) = \frac{\sqrt{2}}{v_0} \,\mathrm{diag}\{m_1, m_2, m_3\}$; 3/ $h^d_{jk} (\equiv \lambda'_{0jk}) = \frac{\sqrt{2}}{v_0} \,\mathrm{diag}\{m_d, m_s, m_b\}$; 4/ $h^u_{ik} = \frac{v_u}{\sqrt{2}} V^\dagger_{CKM} \,\mathrm{diag}\{m_u, m_c, m_t\}$, where $v_0 \equiv \sqrt{2}\, \langle \hat{L}_0 \rangle$ and $v_u \equiv \sqrt{2}\, \langle \hat{H}_u \rangle$. The big advantage here is that the (tree-level) mass matrices for all the fermions do not involve any of the trilinear RPV couplings, even though the approach makes no assumption on any RPV coupling, including those from soft SUSY breaking. Moreover, and all the parameters used are uniquely defined, with the exception of some removable phases. In fact, the (color-singlet) charged fermion mass matrix reduces to the simple form:

$$\mathcal{M}_C = \begin{pmatrix} M_2 & g_2 v_0/\sqrt{2} & 0 & 0 & 0 \\ g_2 v_u/\sqrt{2} & \mu_0 & \mu_1 & \mu_2 & \mu_3 \\ 0 & 0 & m_1 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & 0 & m_3 \end{pmatrix} . \tag{2.2}$$

Each μ_i parameter here characterizes directly the RPV effect on the corresponding charged lepton ($\ell_i = e$, μ, and τ). For any set of other parameter inputs, the m_i's can then be determined, through a simple numerical procedure, to guarantee that the correct mass eigenvalues of m_e, m_μ, and m_τ are obtained — an issue first addressed and solved in ref. [13]. The latter issue is especially important when μ_i's not substantially smaller than μ_0 are considered. Such an odd scenario is not definitely ruled out [13]. However, we would concentrate here on the more popular scenario with only sub-eV neutrino masses and hence small μ_i's. Here deviations of the m_i's from the mass eigenvalues are negligible.

3. Gauginos, higgsinos and neutrinos

The tree-level mixings among the gauginos, higgsinos, and neutrinos gives rise to a 7×7 neutral fermion mass matrix $\mathcal{M}_{\mathcal{N}}$:

$$\mathcal{M}_{\mathcal{N}} = \left(\begin{array}{cccc|ccc} M_1 & 0 & g_1 v_u/2 & -g_1 v_0/2 & 0 & 0 & 0 \\ 0 & M_2 & -g_2 v_u/2 & g_2 v_0/2 & 0 & 0 & 0 \\ g_1 v_u/2 & -g_2 v_u/2 & 0 & -\mu_0 & -\mu_1 & -\mu_2 & -\mu_3 \\ -g_1 v_0/2 & g_2 v_0/2 & -\mu_0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -\mu_1 & 0 & (m_\nu^o)_{11} & (m_\nu^o)_{12} & (m_\nu^o)_{13} \\ 0 & 0 & -\mu_2 & 0 & (m_\nu^o)_{21} & (m_\nu^o)_{22} & (m_\nu^o)_{23} \\ 0 & 0 & -\mu_3 & 0 & (m_\nu^o)_{31} & (m_\nu^o)_{32} & (m_\nu^o)_{33} \end{array}\right), \tag{3.1}$$

whose basis is $(-i\tilde{B}, -i\tilde{W}, \tilde{h}_u^0, \tilde{h}_d^0, \nu_{L_1}, \nu_{L_2}, \nu_{L_3})$, with $\tilde{h}_d^0$ being the neutral fermion from $\hat{L}_0$. The latter is guaranteed to be predominately a neutralino rather than neutrino, as the mass matrix clearly illustrates. As pointed out above, for small μ_i's the charged fermion states in the $\hat{L}_i$'s are essentially the physical states of e, μ and τ. Hence, $(\nu_{L_1}, \nu_{L_2}, \nu_{L_3})$ are essentially ν_e, ν_μ, ν_τ. All entires in the lower-right 3×3 block (m_ν^o) are, of course, zero at tree level. They are induced via 1-loop contributions to be discussed below. Such contributions are the focus of the present study. They are referred to here as direct 1-loop contributions.

We can write the general mass matrix in the form of block submatrices:

$$\mathcal{M}_{\mathcal{N}} = \begin{pmatrix} \mathcal{M}_n & \xi^T \\ \xi & m_\nu^o \end{pmatrix}, \tag{3.2}$$

where $\mathcal{M}_n$ is the upper-left 4×4 neutralino mass matrix, ξ is the 3×4 block, and m_ν^o is the lower-right 3×3 neutrino block in the 7×7 matrix. The resulting (effective) neutrino mass matrix after block diagonalization is given by

$$(m_\nu) = -\xi \mathcal{M}_n^{-1} \xi^T + (m_\nu^o)\,. \tag{3.3}$$

Contributions to the first term here starts at tree level, which are, however, seesaw suppressed. The second term is the direct contribution, which, however, enters in only at 1-loop level. We are interested in the small μ_i scenario, where the tree-level contribution is not necessarily expected to be stronger than such direct 1-loop effects. The 1-loop contributions to the ξ and $\mathcal{M}_n$ blocks are likely to have only a secondary effect on (m_ν). The latter, to be called indirect 1-loop contributions, are not included in the present analysis.

4. LR-mixings for squarks and sleptons

The soft SUSY breaking part of the lagrangian can be written as

$$\begin{aligned}
V_{\rm soft} = {} & \epsilon_{ab} B_\alpha \, H_u^a \tilde{L}_\alpha^b + \epsilon_{ab} \left[A_{ij}^U \, \tilde{Q}_i^a H_u^b \tilde{U}_j^C + A_{ij}^D H_d^a \tilde{Q}_i^b \tilde{D}_j^C + A_{ij}^E H_d^a \tilde{L}_i^b \tilde{E}_j^C \right] + \text{h.c.} + \\
& + \epsilon_{ab} \left[A_{ijk}^{\lambda'} \tilde{L}_i^a \tilde{Q}_j^b \tilde{D}_k^C + \frac{1}{2} \, A_{ijk}^{\lambda} \tilde{L}_i^a \tilde{L}_j^b \tilde{E}_k^C \right] + \frac{1}{2} \, A_{ijk}^{\lambda''} \tilde{U}_i^C \tilde{D}_j^C \tilde{D}_k^C + \text{h.c.} + \\
& + \tilde{Q}^\dagger \tilde{m}_Q^2 \, \tilde{Q} + \tilde{U}^\dagger \tilde{m}_U^2 \, \tilde{U} + \tilde{D}^\dagger \tilde{m}_D^2 \, \tilde{D} + \tilde{L}^\dagger \tilde{m}_L^2 \tilde{L} + \tilde{E}^\dagger \tilde{m}_E^2 \, \tilde{E} + \tilde{m}_{H_u}^2 \, |H_u|^2 + \\
& + \frac{M_1}{2} \tilde{B} \tilde{B} + \frac{M_2}{2} \tilde{W} \tilde{W} + \frac{M_3}{2} \tilde{g} \tilde{g} + \text{h.c.} \,,
\end{aligned} \tag{4.1}$$

where we have separated the R-parity conserving A-terms from the RPV ones (recall $\hat{H}_d \equiv \hat{L}_0$). Note that $\tilde{L}^\dagger \tilde{m}_{\tilde{L}}^2 \tilde{L}$, unlike the other soft mass terms, is given by a 4×4 matrix. Explicitly, $\tilde{m}_{L_{00}}^2$ corresponds to $\tilde{m}_{H_d}^2$ of the MSSM case while $\tilde{m}_{L_{0k}}^2$'s give RPV mass mixings.

The SVP also simplifies much the otherwise complicated expressions for the mass-squared matrix of the scalar sectors. Firstly, we will look at the squark sectors. The masses of up-squarks obviously have no RPV contribution. The down-squark sector, however, has an interesting result. We have the mass-squared matrix as follows :

$$\mathcal{M}_D^2 = \begin{pmatrix} \mathcal{M}_{LL}^2 & \mathcal{M}_{RL}^{2\dagger} \\ \mathcal{M}_{RL}^2 & \mathcal{M}_{RR}^2 \end{pmatrix}, \tag{4.2}$$

where

$$\begin{aligned}
\mathcal{M}_{LL}^2 &= \tilde{m}_Q^2 + m_D^\dagger m_D + M_Z^2 \, \cos 2\beta \left[-\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right], \\
\mathcal{M}_{RR}^2 &= \tilde{m}_D^2 + m_D m_D^\dagger + M_Z^2 \, \cos 2\beta \left[-\frac{1}{3} \sin^2 \theta_W \right],
\end{aligned} \tag{4.3}$$

and

$$\left(\mathcal{M}_{RL}^2 \right)^T = A^D \frac{v_0}{\sqrt{2}} - m_D \, \mu_0^* \, \tan \beta - \left(\mu_i^* \lambda'_{ijk} \right) \frac{v_u}{\sqrt{2}} \,. \tag{4.4}$$

Here, m_D is the down-quark mass matrix, which is diagonal under the parametrization adopted; $(\,\mu_i^* \lambda'_{ijk}\,)$ denotes the 3×3 matrix $(\;\;)_{jk}$ with elements listed; and $\tan \beta = v_u / v_0$. Note that all the VEV's can be taken as real, so long as the tree level scalar potential is considered (see the appendix). Apart from the first A^D term, the remaining terms in $(\mathcal{M}_{RL}^2)^T$ are F-term contributions; in particular, the last term gives "SUSY conserving" but R-parity violating contributions; note that the existence of non-zero F-terms or electroweak symmetry breaking VEV's can be interpreted as a consequence of SUSY breaking though. The full F-term part in the above equation can actually be written together as $(\,\mu_\alpha^* \lambda'_{\alpha jk}) \, \frac{v_u}{\sqrt{2}}$ where the $\alpha = 0$ term, which vanishes for $j \neq k$, gives the second term in the r.h.s. The latter, of course, is just the usual μ-term contribution in the MSSM case.

Next we move on to the slepton sector. From eq. (4.1) above, we can see that the "charged Higgs" should be considered together with the sleptons. We have hence an 8×8 mass-squared matrix of the following $1+4+3$ form:

$$\mathcal{M}_E^2 = \begin{pmatrix} \widetilde{\mathcal{M}}_{Hu}^2 & \widetilde{\mathcal{M}}_{LH}^{2\dagger} & \widetilde{\mathcal{M}}_{RH}^{2\dagger} \\ \widetilde{\mathcal{M}}_{LH}^2 & \widetilde{\mathcal{M}}_{LL}^2 & \widetilde{\mathcal{M}}_{RL}^{2\dagger} \\ \widetilde{\mathcal{M}}_{RH}^2 & \widetilde{\mathcal{M}}_{RL}^2 & \widetilde{\mathcal{M}}_{RR}^2 \end{pmatrix}, \tag{4.5}$$

where

$$\begin{aligned} \widetilde{\mathcal{M}}_{LL}^2 &= \tilde{m}_L^2 + m_L^\dagger m_L + (\mu_\alpha^* \mu_\beta) + M_Z^2 \cos 2\beta \left[-\frac{1}{2} + \sin^2\theta_W \right] + \\ &\quad + \begin{pmatrix} M_Z^2 \cos^2\beta [1 - \sin^2\theta_W] & 0_{1\times 3} \\ 0_{3\times 1} & 0_{3\times 3} \end{pmatrix}, \\ \widetilde{\mathcal{M}}_{RR}^2 &= \tilde{m}_E^2 + m_E m_E^\dagger + M_Z^2 \cos 2\beta \left[-\sin^2\theta_W \right], \\ \widetilde{\mathcal{M}}_{Hu}^2 &= \tilde{m}_{H_u}^2 + \mu_\alpha^* \mu_\alpha + M_Z^2 \cos 2\beta \left[\frac{1}{2} - \sin^2\theta_W \right] + \\ &\quad + M_Z^2 \sin^2\beta \left[1 - \sin^2\theta_W \right]; \end{aligned} \tag{4.6}$$

and

$$\left(\widetilde{\mathcal{M}}_{RL}^2 \right)^T = \begin{pmatrix} 0 \\ A^E \end{pmatrix} \frac{v_0}{\sqrt{2}} - \begin{pmatrix} 0 \\ m_E \end{pmatrix} \mu_0^* \tan\beta - (\mu_i^* \lambda_{i\beta k}) \frac{v_u}{\sqrt{2}}, \tag{4.7}$$

$$\widetilde{\mathcal{M}}_{RH}^2 = -(\mu_i^* \lambda_{i0k}) \frac{v_0}{\sqrt{2}}, \tag{4.8}$$

$$\widetilde{\mathcal{M}}_{LH}^2 = (B_\alpha^*) + \begin{pmatrix} \frac{1}{2} M_Z^2 \sin 2\beta [1 - \sin^2\theta_W] \\ 0_{3\times 1} \end{pmatrix}. \tag{4.9}$$

Here, $m_L = \mathrm{diag}\{0, m_E\} \equiv \mathrm{diag}\{0, m_1, m_2, m_3\}$, where the three m_i's are masses from leptonic Yukawa terms as discussed above in relation to eq. (2.2); and, again, $(\mu_i^* \lambda_{i\beta k})$ denotes a matrix (4×3) with elements given by $(\;)_{\beta k}$. Recall that for the small μ_i domain we focused on here in this paper, we have $m_E \simeq \mathrm{diag}\{m_e, m_\mu, m_\tau\}$. In fact, the k-th element in the 3-column-vector $\widetilde{\mathcal{M}}_{RH}^2$ in eq. (4.8) can be written as simply as $\mu_k^* m_k$ (no sum). Similarly, the k-th element in the first row of the 4×3 matrix $(\widetilde{\mathcal{M}}_{RL}^2)^T$ in eq. (4.7) can be written as $\mu_k^* m_k \tan\beta$ (no sum). The former is a $\tilde{\ell}_R^c h_u^-$ type, while the latter a $\tilde{\ell}_R^c h_d^-$ type ($h_d^- \equiv \tilde{\ell}_{L_0}$), mass-squared term. Or, to better illustrate the common flavor structure, one can put the full F-term part of eq. (4.7) as $-(\mu_\alpha^* \lambda_{\alpha\beta k}) \frac{v_u}{\sqrt{2}}$.

For the sake of completeness, we also give explicitly the neutral scalar, (or sneutrino-Higgs) mass-squared matrix. The neutral scalar mass terms, in terms of the $(1+4)$ complex scalar fields, ϕ_n's, can be written in two parts — a simple $(\mathcal{M}_\phi^2)_{mn}\, \phi_m^\dagger \phi_n$ part, and a Majorana-like part in the form $\frac{1}{2}(\mathcal{M}_{\phi\phi}^2)_{mn}\, \phi_m \phi_n + \mathrm{h.c.}$. As the neutral scalars are originated from chiral doublet superfields, the existence of the Majorana-like part is a direct consequence of the electroweak symmetry breaking VEV's, hence restricted to the scalars playing the Higgs role only. They come from the quartic terms of the Higgs fields in the scalar potential. We have explicitly

$$\mathcal{M}^2_{\phi\phi} = \frac{1}{2}\, M_Z^2 \begin{pmatrix} \sin^2\beta & -\cos\beta\,\sin\beta & 0_{1\times 3} \\ -\cos\beta\,\sin\beta & \cos^2\beta & 0_{1\times 3} \\ 0_{3\times 1} & 0_{3\times 1} & 0_{3\times 3} \end{pmatrix} ; \tag{4.10}$$

and

$$\mathcal{M}^2_{\phi} = \begin{pmatrix} \tilde{m}^2_{H_u} + \mu^*_\alpha \mu_\alpha + M_Z^2\, \cos 2\beta\, [-1/2] & -(B_\alpha) \\ -(B^*_\alpha) & \tilde{m}^2_L + (\mu^*_\alpha \mu_\beta) + M_Z^2\, \cos 2\beta\, [-1/2] \end{pmatrix} + \\ + \mathcal{M}^2_{\phi\phi}\, . \tag{4.11}$$

Note that $\mathcal{M}^2_{\phi\phi}$ here is real (see the appendix), while $\mathcal{M}^2_{\phi}$ does have complex entries. The full 10×10 (real and symmetric) mass-squared matrix for the real scalars is then given by

$$\mathcal{M}^2_S = \begin{pmatrix} \mathcal{M}^2_{SS} & \mathcal{M}^2_{SP} \\ (\mathcal{M}^2_{SP})^T & \mathcal{M}^2_{PP} \end{pmatrix} , \tag{4.12}$$

where the scalar, pseudo-scalar, and mixing parts are

$$\begin{aligned} \mathcal{M}^2_{SS} &= \mathrm{Re}(\mathcal{M}^2_{\phi}) + \mathcal{M}^2_{\phi\phi}\, , \\ \mathcal{M}^2_{PP} &= \mathrm{Re}(\mathcal{M}^2_{\phi}) - \mathcal{M}^2_{\phi\phi}\, , \\ \mathcal{M}^2_{SP} &= -2\, \mathrm{Im}(\mathcal{M}^2_{\phi})\, , \end{aligned} \tag{4.13}$$

respectively. If $\mathrm{Im}(\mathcal{M}^2_{\phi})$ vanishes, the scalars and pseudo-scalars decouple from one another and the unphysical Goldstone mode would be found among the latter. Finally, we note that the B_α entries may also be considered as a kind of LR mixings. The RPV B_i's do in fact contribute to neutrino mass, as discussed below.

We would like to emphasize that the above scalar mass results are complete — all RPV contributions, SUSY breaking or otherwise, are included without theoretical bias. The simplicity of the result is a consequence of the SVP. Explicitly, there are no RPV A-term contributions due to the vanishing of VEV's $v_i \equiv \sqrt{2}\langle \hat{L}_i \rangle$. However, such new contributions, as well as their roles in the physics of neutrino masses and phenomena like fermion EDM's and $b \to s\,\gamma$, are genuine. For instance, rotating to a basis among the $\hat{L}_\alpha$ superfields under which the μ_i's are zero would restore the $\hat{L}_i$ VEV's and show e.g. the RPV $(\,\mu^*\lambda\,)$ term as a term involving the latter VEV's and some A-term parameters. The Higgs-slepton results given as in eqs. (4.5) and (4.11) are admittedly not very useful for doing scalar physics. They contain a redundancy of parameters and hide the unphysical Goldstone state. However, for the purpose of analyzing the neutrino mass contributions as done below, they serve their purpose. Hence, we will refrain from further laboring on the algebra here.

Before ending the section, we want to emphasize the following. While it should be straight forward to write down the scalar mass matrices by the complete theory of SUSY without R-parity, under any formulation or parametrization, to the best of our knowledge, this has not been published before. After completing the work, we checked the literature and found no explicitly written down complete results for $\mathcal{M}_D^2$ and $\mathcal{M}_E^2$ as given here. Especially the existence of the interesting new RPV contributions, of the type given by the $(\,\mu^*\lambda'\,)$ term in $\mathcal{M}_D^2$, and the $(\,\mu^*\lambda\,)$ term in eq. (4.7), and the $\mu^*\mu$ terms in eqs. (4.5) and (4.11), if noticed, have not been much appreciated. Explicit discussion of terms of the type $(\,\mu^*\lambda\,)$ and their contribution to neutrino masses was first given in a recent paper by K. Cheung and the present author [9], in a different context. Their existence and important phenomenological implication seems, otherwise, to have been overlooked.

We are not aware of any other explicit discussion of the $(\,\mu^*\lambda'\,)$ and $(\,\mu^*\lambda\,)$ terms in the literature. What follows is a check into the literature for the scalar mass results. Before doing that, however, we would like to emphasize that the complete theory of SUSY without R-parity is not a very popular subject, compared to various versions of more specific (assumed) forms of R-parity violation. We also warn the readers that most of the previous authors were not working under the parametrization we used here. In our opinion, there are many other subtle complications when a different parametrization is used, which have not been explicitly addressed. The interested readers are referred to a forthcoming review by the author on the subject [19]. The complete scalar mass results in a generic $\hat{L}_\alpha$ flavor basis would look more complicated than what we have here too. In particular, there would be contributions involving the trilinear RPV A-terms. Having said that, let us take a look at some works on a more or less complete version of R-parity violation and the scalar mass expression given therein. Ref. [20] gives "sfermion mass matrix" exactly as in the MSSM without RPV terms at all. Ref. [12] is a more careful and detailed study. However, as mentioned above, the paper considers only one λ and one λ' and hence does not give result in the complete theory anyway. An admissible $(\mu^*\lambda')$ term is still missing in the down-squark mass-squared matrix given. An 8×8 matrix corresponding to $\mathcal{M}_E^2$ is indeed presented. The matrix seems correct, under the starting assumption, though the admissible $(\,\mu^*\lambda\,)$ term is not explicitly shown. Ref. [3] is the first to give the matrix in the 8×8 form, but we find no clear sign of the $(\,\mu^*\lambda\,)$ term. The squark mass-squared matrix is not explicitly given there. In fact, the paper is based on a specific high energy scenario, which is hence not totally generic. For instance, early in the paper, the soft SUSY breaking part of the lagrangian is already given in a simplified form with the high energy assumptions put in, hence not for the generic complete theory we discussed here. The neutral scalar mass, corresponding to $\mathcal{M}_S^2$ above, is better known [21]. In particular, ref. [10] gives the result essentially under the SVP, though truncated to one lepton family.

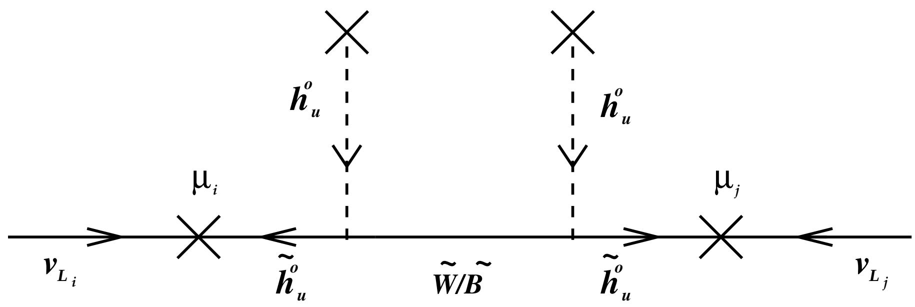

Figure 1: Neutrino mass from tree-level seesaw.

Another important point to note is that we have pay special attention to the fact there the RPV parameters are generally complex, while this phase information has not been explicitly given previously. As mentioned above, the λ' term, for example, contributes to neutron EDM through its imaginary part. The other example is the existence of scalar-pseudo-scalar mixing as a result of complex RPV parameters, as given above explicitly in $\mathcal{M}^2_{SP}$. The significant phenomenological implication of the latter has been well illustrated in ref. [22], in the case of MSSM, for instance.

Finally, we note that there is another group that has done quite elaborated works on their RPV model (see for example ref. [6]), which however includes no trilinear RPV parameters and hence would not have our results for the complete theory. Moreover, in the soft SUSY breaking part given and used in ref. [6] actually has $\tilde{m}^2_{L_{0k}}$ term missing.

We have not actually done a detailed term by term checking to see if there are other discrepancies between ours results given in this section and others in the papers mentioned. The above brief comparison is supposed to serve as an illustration of the point we want to make here — that perhaps insufficient care and attention have been given to the complete results for the scalar masses in SUSY without R-parity. We will report more on the various phenomenological implications of some of the RPV mass entries in some other publications.

5. Neutrino mass contributions

Let us return to RPV contribution to neutrino mass. From eq. (3.1), one neutrino state get a tree-level mass. The seesaw suppressed contribution [see eqs. (3.2) and (3.3)] is given by

$$(m_\nu)^{\text{tree}}_{ij} \sim \frac{-v^2\cos^2\beta\,(g_2^2 M_1 + g_1^2 M_2)}{2\mu_0\left[2\mu_0 M_1 M_2 - v^2 \sin\beta\cos\beta\,(g_2^2 M_1 + g_1^2 M_2)\right]}\mu_i\mu_j\,. \tag{5.1}$$

This is illustrated diagrammatically in figure 1.

Next we come to the direct 1-loop contributions. A typical 1-loop neutrino mass diagram has two couplings of scalar-fermion-neutrino type. With the two couplings

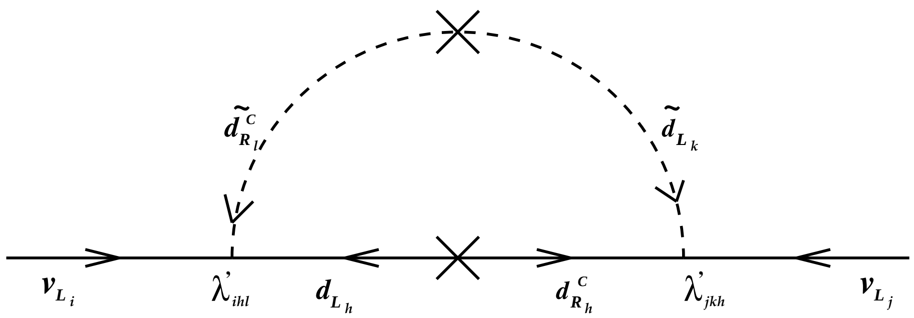

Figure 2: Neutrino mass from quark-squark loop.

being λ'-type, we have a quark-squark loop as shown in figure 2. Here, a LR squark mixing is needed. From eqs. (4.2) and (4.4), we have the result, here written in three parts : firstly the familiar one

$$(m_\nu)^{\rm sqA}_{ij} \sim \frac{3}{16\pi^2}\,\frac{m_{D_h} m_{D_k}}{M^2_{\tilde{d}}}\,\lambda'_{ihk}\lambda'_{jkh}\,[A_d - \mu_0^*\,\tan\beta] \quad (i \longleftrightarrow j)\,, \tag{5.2}$$

where $M_{\tilde{d}}$ denote an average down-squark mass, and A_d being a constant (mass) parameter representing the "proportional" part of the A-term, namely $A^D \frac{v_0}{\sqrt{2}} = A_d\, m_D + \delta A^D\ \frac{v_0}{\sqrt{2}}$, and m_{D_h} is the h-th diagonal element of the matrix m_D (i.e. the quark mass); next, the "proportionality" violating part

$$(m_\nu)^{\rm sq\delta A}_{ij} \sim \frac{3}{16\pi^2}\,\frac{m_{D_h}}{M^2_{\tilde{d}}}\,\lambda'_{ihl}\lambda'_{jkh}\left[\delta A^D_{kl}\frac{v_0}{\sqrt{2}}\right] \quad (i \longleftrightarrow j)\,, \tag{5.3}$$

which is typically expected to be suppressed in many SUSY breaking scenarios and neglected; and, finally, the part due to the new RPV LR mixings,

$$(m_\nu)^{{\rm sq}\not{R}}_{ij} \sim -\frac{3}{16\pi^2}\,\frac{m_{D_h}}{M^2_{\tilde{d}}}\lambda'_{ihl}\lambda'_{jkh}\ \left[\mu^*_g\lambda'_{gkl}\ \frac{v_u}{\sqrt{2}}\right] \quad (i \longleftrightarrow j)\,. \tag{5.4}$$

The $(i \leftrightarrow j)$ expression denote symmetrization with respect to i and j. It is interesting to note that the last result contains no SUSY breaking parameter in the LR mixings. In particular, the flavor changing parts of the latter could not be suppressed through any SUSY breaking mechanism.

Similar to the quark-squark loop, a lepton-slepton loop with two λ-type coupling, as shown in figure 3, generates neutrino mass, in the presence of LR slepton mixings. Using eqs. (4.5) and (4.7), again we split the result into the different parts : the familiar one from the "proportional" part of the A-term,

$$(m_\nu)^{\rm slA}_{ij} \sim \frac{1}{16\pi^2}\,\frac{m_h m_k}{M^2_{\tilde{\ell}}}\,\lambda_{ihk}\lambda_{jkh}\,[A_e - \mu_0^*\,\tan\beta] \quad (i \longleftrightarrow j)\,, \tag{5.5}$$

where $M_{\tilde{\ell}}$ denote an average charged slepton mass, and A_e the constant (mass) parameter with $A^E \frac{v_0}{\sqrt{2}} = A_e\, m_E + \delta A^E\ \frac{v_0}{\sqrt{2}}$, (recall that m_h's are diagonal element of m_E

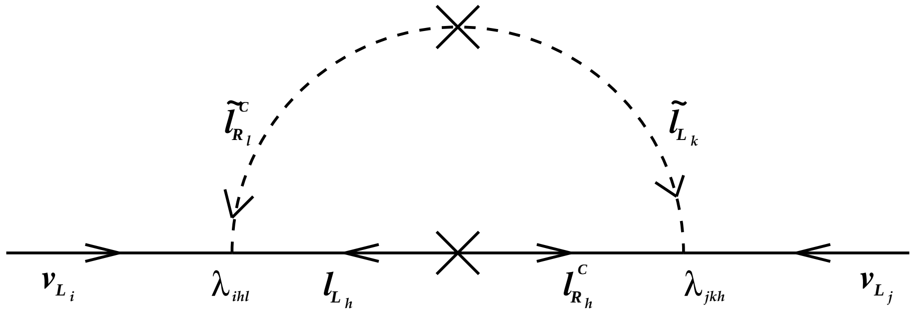

Figure 3: Neutrino mass from lepton-slepton loop.

and essentially the mass of the charged lepton); the "proportionality" violating part

$$(m_\nu)^{\mathrm{sl}\delta\mathrm{A}}_{ij} \sim \frac{1}{16\pi^2}\,\frac{m_h}{M^2_{\tilde{\ell}}}\,\lambda_{ihl}\lambda_{jkh}\,\left[\delta A^E_{kl}\frac{v_0}{\sqrt{2}}\right] \quad (i \longleftrightarrow j)\,; \tag{5.6}$$

and the part due to the new RPV LR mixings,

$$(m_\nu)^{\mathrm{sl}\not{R}}_{ij} \sim -\frac{1}{16\pi^2}\,\frac{m_h}{M^2_{\tilde{\ell}}}\,\lambda_{ihl}\lambda_{jkh}\,\left[\mu^*_g\lambda_{gkl}\,\frac{v_u}{\sqrt{2}}\right] \quad (i \longleftrightarrow j)\,. \tag{5.7}$$

However, the above is not yet the full result for the type of contributions. We have emphasized throughout the paper the systematic treatment of making no *a priori* distinction between the $\hat{L}_i$'s and $\hat{H}_d$. The latter is denoted as $\hat{L}_0$ and treated as a 4-th leptonic flavor. In eq. (4.7), the last term admitted a $\beta = 0$ part the neutrino mass contribution of which has not been included in the above analysis of the lepton-slepton loop parallel to the quark-squark loop. The corresponding result is simply given by setting k to 0 in eq. (5.7), which may then be simplified to

$$(m_\nu)^{\mathrm{slZ}}_{ij} \sim -\frac{1}{16\pi^2}\,\frac{\sqrt{2}}{v_0}\,\frac{m_j^2}{M^2_{\tilde{\ell}}}\,\lambda_{ijl}\,[\mu^*_l m_l\,\tan\beta] \quad (i \longleftrightarrow j)\,. \tag{5.8}$$

The contribution corresponds to the SUSY analog of the Zee neutrino mass diagram [23], as discussed in ref. [9]. We illustrate the contribution and its Zee model analog in figure 4. A careful examination of figure 3 shows that one cannot get any more new neutrino mass diagram by replacing some other λ_{ijk} flavor indices with a 0. Hence, we have completed the listing of the two-λ-loop contributions.

The above has exhausted all the possibilities from using from the trilinear superpotential couplings as the only source for the loop vertices. The only other couplings involving a neutrino are the gauge couplings and the bilinear μ_i's. The effect of the latter has been considered in the tree-level seesaw. Putting two gauge couplings together, we do have a 1-loop neutrino mass diagram, with scalars and gauginos running in the loop. The charged loop does not work, while a neutral loop could do (see figure 5) when there is a Majorana-like sneutrino mass term. The latter

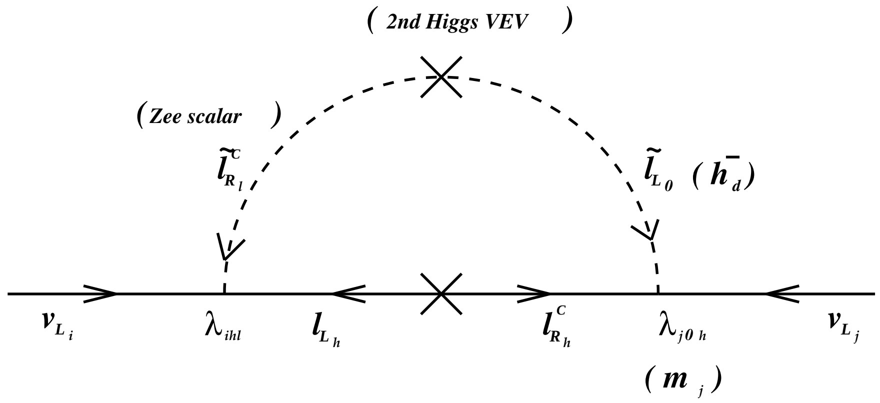

Figure 4: SUSY Zee diagram for neutrino mass. The Zee analog interpretation noted in the brackets.

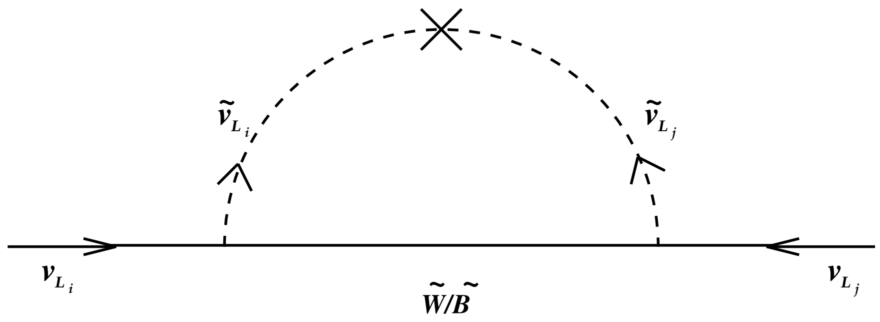

Figure 5: Gaugino-sneutrino loop requiring a Majorana-like sneutrino mass insertion.

contribution was first pointed out in ref. [10]. In fact, Majorana-like sneutrino mass is where the required two units of lepton number violation come in. The former may be interpreted as a result of splitting in mass of the sneutrino and anti-sneutrino due to R-parity violation. Following our general approach here, we illustrate this in figure 6. It is clear from the figure that it involves the SUSY breaking and RPV parameters B_i's, as shown is eq. (4.11) above, and is seesaw suppressed (cf. figure 1), unless the B_i's happen to be at the SUSY scale despite small μ_i's. Note that the B_i's and the μ_i's are not totally independent parameters, as illustrated in the appendix. It is very unlikely that B_i/B_0 would be much larger than μ_i/μ_0.

The gauge loop contribution discussed serves as an illustrative example of what we call pseudo-direct 1-loop. The EW state diagram as given in figure 5 reads zero, as the required mass insertion on the sneutrino line does not exist [cf. eq. (4.10)]. If we admit extra mass insertions and extend the sneutrino line as shown in figure 6, we obtain the non-zero result. However, one show bear in mind that there is no definite hierarchy between this gauge loop contribution and the other direct 1-loop ones discussed above, as they arise from different RPV parameters, the magnitude of which we have no exact information.

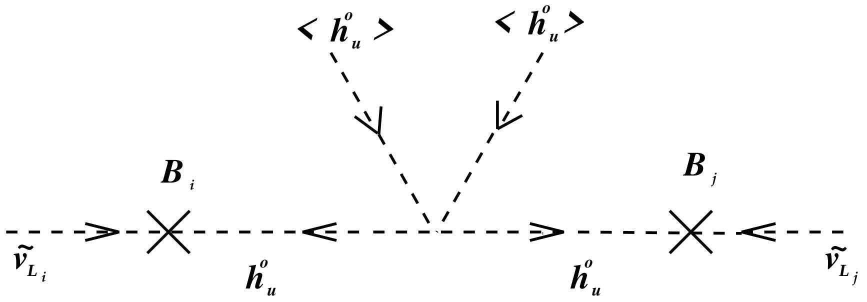

Figure 6: Seesaw diagram for Majorana-like sneutrino masses.

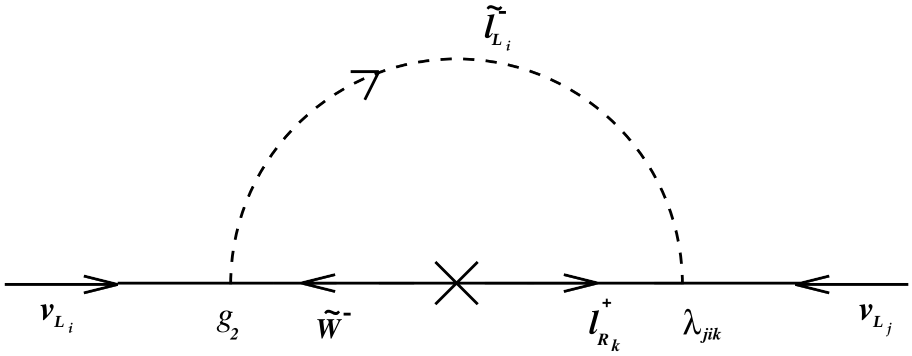

Figure 7: A direct charged loop contribution requiring however a fermion mass insertion that is vanishing.

Finally, it is not difficult to see that there is no contribution from 1-loop diagrams with one gauge coupling and one Yukawa or λ-coupling vertices, up to the direct 1-loop level. In fact, before we put in non-minimal number of mass insertions in the internal lines, there is only one such EW state diagram, as given in figure 7. The diagram requires a $\tilde{W}^-$-$\ell^c_{R_k}$ mass insertion, which is zero under the SVP. When we admit extra mass insertions along the internal fermion line and go to the pseudo-direct 1-loop level, there are apparent non-zero contribution. Obviously we need at least one lepton number violating mass insertion. Figure 8 illustrates the minimal extra mass insertions along the fermion line that could complete a diagram.

However, let us look more closely into the implications of putting in extra mass insertions along the internal fermion line. The scalar-fermion loop neutrino mass diagram result always has a mass factor of the internal fermion in it; explicit examples are m_{D_h} in figure 2 and m_h in figure 3. To obtain the exact result, one should use the mass eigenstates, and of course sum over the latter (see, for example, formulae in ref. [12]). However, neglecting the fermion mass dependence in the propagator integral part, the mass and mixing matrix element dependence of the full sum is of course nothing other than the tree-level mass entry of single insertion case, i.e. the vanishing $\tilde{W}^-$-$\ell^c_{R_k}$ term for the case at hand. If one consider only the contribution

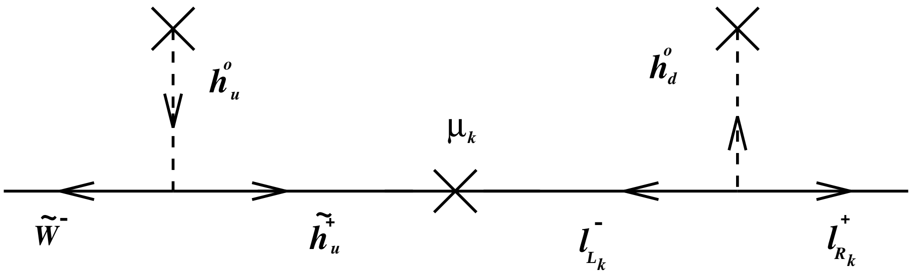

Figure 8: A possible set of mass insertions apparently producing the required internal fermion line in the previous figure.

from one of the mass eigenstates, the result would not be zero. But we know that it is going to be canceled by that from the other mass eigenstates. This is like the GIM mechanism, violated here only to the extent of the non-universal mass effect from the propagator integral part.

In terms of EW state diagrams, the exact mass eigenstate result would correspond to summing over all possible diagrams with any (up to infinite) number of admissible mass insertions. The contribution from putting figure 8 into the internal fermion line of figure 7 is of course just one among the non-trivial result. Hence, taking this as an independent contribution is more or less equivalent to taking one term from the summation over mass eigenstates. The result of the latter is expected to be canceled in the overall sum.

Furthermore, even if that GIM-like cancellation is rendered ineffective by the propagator integral part, the dependence of the contribution on a single λ_{jik} implies that upon symmetrization with respect to i and j, there would be another cancellation as from $\lambda_{ijk} = -\lambda_{jik}$ to the extent that $\tilde{\ell}_{L_j}^-$ and $\tilde{\ell}_{L_i}^-$ has the same mass. So, even the naive pseudo-direct 1-loop contribution has suppression from the expected degeneracy of slepton masses, and is only proportional to the violation of the latter.

This, together with the gauge loop discussed above, has exhausted our discussion of the pseudo-direct 1-loop contributions. One could certainly get other EW state diagrams by putting in extra mass insertion, into the scalar line of figures 5 or 7. While direct 1-loop results from an EW state diagram roughly represent the corresponding exact mass eigenstate results, pseudo-direct 1-loop diagrams may be just pieces of an otherwise small or vanishing overall sum with a GIM-like cancellation mechanism at work. We do have to go to exact numerical calculations to know the extent to which the latter is violated and extract the correct result. Perhaps it should also be said that we have not considered diagrams with mass insertion(s) on the external lines because such diagrams really correpond the indirect 1-loop contributions. A diagram with one mass insertion in one of the ν external line, for example, should correspond to something in the ξ block of eq. (3.1). We want to emphasize that while we classified contributions into direct 1-loop, pseudo-direct 1-loop and indirect

1-loop, there is no definite hierarchy among them, when one is comparing contributions involving different RPV parameters. A clear example is given by the fact that we do not know if the tree-level contribution which involves the μ_i's is really larger than, for example a λ' (quark-squark) loop contribution, though one may naively expect so.

6. Concluding remarks

From the above systematic analysis, it is clear that we have discussed and given explicit formulae for all neutrino mass contributions up to the level of direct 1-loop contribution, for the complete theory of SUSY without R-parity. Psuedo-direct 1-loop contributions are also discussed. We have also given a description of the full squark and slepton masses. The latter is useful for analyzing other aspects of phenomenology, particularly those related to LR mixings such as fermion electric dipole moment and flavor changing neutral current processes. The successful simple description here illustrates well the effectiveness of the formulation (SVP) adopted.

Note added. After we posted the first version of this paper, a paper from Davidson and Losada [24] on the subject appeared. The paper does list results under our formulation (SVP) here and goes beyond direct or pseudo-direct 1-loop level. In particular, a nice discussion of the gauge loop result is included. However, the basic approach is very different from that of this paper, and the contributions from the RPV LR scalar mixing are not included. Comparing our results with theirs, there seems to be some apparent disagreements, which we hope to address in detail in a future publication.

Acknowledgments

The author is in debt to S.K. Kang and K. Cheung for discussions and for suggestions on improving the present manuscript, and to A. Akeroyd for proof-reading the last version and helps to improve the language. S. Davidson and M. Losada are to be thanked for communications concerning their recent paper [24]. The Korea Institute for Advanced Study is to be thanked for their hospitality during the early phase of our work. The Yukawa Institute for Theoretical Phyiscs, Kyoto University, and KEK, where the last revision of the manuscript is done, is to be thanked likewise.

A. Note on the scalar potential

In terms of the five, plausibly electroweak symmetry breaking, neutral scalars fields ϕ_n, the generic (tree-level) scalar potential, as constrained by SUSY, can be written

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as:

$$V_s = Y_n \left|\phi_n\right|^4 + X_{mn} \left|\phi_m\right|^2 \left|\phi_n\right|^2 + \hat{m}_n^2 \left|\phi_n\right|^2 - \\ - \left(\hat{m}_{mn}^2 e^{i\theta_{mn}} \phi_m^\dagger \phi_n + \text{h.c.}\right) \quad (m < n) . \tag{A.1}$$

Here, we count the ϕ_n's from -1 to 3 and identify a ϕ_α (recall $\alpha = 0$ to 3) as $\tilde{l}_\alpha^0$ and $\phi_{\text{-}1}$ as h_u^0. Parameters in the above expression for V_s (all real) are then given by

$$\begin{aligned} \hat{m}_\alpha^2 &= \tilde{m}_{L_{\alpha\alpha}}^2 + \left|\mu_\alpha\right|^2 , \\ \hat{m}_{\text{-}1}^2 &= \tilde{m}_{H_u}^2 + \mu_\alpha^* \mu_\alpha , \\ \hat{m}_{\alpha\beta}^2 \, e^{i\theta_{\alpha\beta}} &= -\tilde{m}_{L_{\alpha\beta}}^2 - \mu_\alpha^* \mu_\beta \quad (\text{no sum}) , \\ \hat{m}_{\text{-}1\alpha}^2 \, e^{i\theta_{\text{-}1\alpha}} &= B_\alpha \quad (\text{no sum}) , \\ Y_n &= \frac{1}{8}(g_1^2 + g_2^2) , \\ X_{\text{-}1\alpha} &= -\frac{1}{4}(g_1^2 + g_2^2) = -X_{\alpha\beta} . \end{aligned} \tag{A.2}$$

Under the SVP, we write the VEV's as follows:

$$\begin{aligned} v_{\text{-}1} \left(\equiv \sqrt{2}\, \langle \phi_{\text{-}1} \rangle\right) &= v_u , \\ v_0 \left(\equiv \sqrt{2}\, \langle \phi_0 \rangle\right) &= v_d \, e^{i\theta_v} , \\ v_i \left(\equiv \sqrt{2}\, \langle \phi_i \rangle\right) &= 0 , \end{aligned} \tag{A.3}$$

where we have put in a complex phase in the VEV v_0, for generality.

The equations from the vanishing derivatives of V_s along $\phi_{\text{-}1}$ and ϕ_0 give

$$\begin{aligned} \left[\frac{1}{8}(g_1^2 + g_2^2)(v_u^2 - v_d^2) + \hat{m}_{\text{-}1}^2\right] v_u &= B_0 \, v_d \, e^{i\theta_v} , \\ \left[\frac{1}{8}(g_1^2 + g_2^2)(v_d^2 - v_u^2) + \hat{m}_0^2\right] v_d &= B_0 \, v_u \, e^{i\theta_v} . \end{aligned} \tag{A.4}$$

Hence, $B_0 \, e^{i\theta_v}$ is real. In fact, the part of V_s that is relevant to obtaining the tadpole equations is no different from that of MSSM apart from the fact that $\tilde{m}_{H_u}^2$ and $\tilde{m}_{H_d}^2$ of the latter are replaced by $\hat{m}_{\text{-}1}^2$ and $\hat{m}_0^2$, respectively. As in MSSM, the B_0 parameter can be taken as real. The conclusion here is therefore that θ_v vanishes, or all VEV's are real, despite the existence of complex parameters in the scalar potential. Results from the other tadpole equations, in a ϕ_i direction, are quite simple. They can be written as complex equations of the form

$$\hat{m}_{\text{-}1i}^2 \, e^{i\theta_{\text{-}1i}} \tan\beta = -e^{i\theta_v} \hat{m}_{0i}^2 \, e^{i\theta_{0i}} , \tag{A.5}$$

which is equivalent to

$$B_i \, \tan\beta = \tilde{m}_{L_{0i}}^2 + \mu_0^* \mu_i , \tag{A.6}$$

where we have used $v_u = v \sin\beta$ and $v_d = v\cos\beta$. Note that our $\tan\beta$ has the same physical meaning as that in the R-parity conserving case. For instance, $\tan\beta$, together with the corresponding Yukawa coupling ratio, gives the mass ratio between the top and the bottom quark.

The three complex equations for the B_i's reflect the redundance of parameters in a generic $\hat{L}_\alpha$ flavor basis. The equations also suggest that the B_i's are expected to be suppressed, with respect to the R-parity conserving B_0, as the μ_i's are, with respect to μ_0. They give consistence relationships among the involved RPV parameters (under the SVP) that should not be overlooked.

References

[1] For an updated review, see V. Barger, *Neutrino masses and mixing at the millennium*, hep-ph/0003212.

[2] L.J. Hall and M. Suzuki, *Explicit R-parity breaking in supersymmetric models*, *Nucl. Phys.* **B 231** (1984) 419.

[3] R. Hempfling, *Neutrino masses and mixing angles in SUSY-GUT theories with explicit R-parity breaking*, *Nucl. Phys.* **B 478** (1996) 3 [hep-ph/9511288].

[4] H.-P. Nilles and N. Polonsky, *Supersymmetric neutrino masses, R-symmetries and the generalized μ problem*, *Nucl. Phys.* **B 484** (1997) 33 [hep-ph/9606388].

[5] M. Drees, S. Pakvasa, X. Tata and T. ter Veldhuis, *A supersymmetric resolution of solar and atmospheric neutrino puzzles*, *Phys. Rev.* **D 57** (1998) 5335 [hep-ph/9712392];
V. Bednyakov, A. Faessler and S. Kovalenko, SUPER-KAMIOKANDE *constraints on R-parity violating supersymmetry*, *Phys. Lett.* **B 442** (1998) 203 [hep-ph/9808224];
B. Mukhopadhyaya, S. Roy and F. Vissani, *Correlation between neutrino oscillations and collider signals of supersymmetry in an R-parity violating model*, *Phys. Lett.* **B 443** (1998) 191 [hep-ph/9808265];
S. Rakshit, G. Bhattacharyya and A. Raychaudhuri, *R-parity-violating trilinear couplings and recent neutrino data*, *Phys. Rev.* **D 59** (1999) 091701 [hep-ph/9811500];
A.S. Joshipura and S.K. Vempati, *Gauge mediated supersymmetry breaking and neutrino anomalies*, *Phys. Rev.* **D 60** (1999) 095009 [hep-ph/9808232];
E.J. Chun, S.K. Kang, C.W. Kim and U.W. Lee, *Supersymmetric neutrino masses and mixing with R-parity violation*, *Nucl. Phys.* **B 544** (1999) 89 [hep-ph/9807327];
F. Takayama and M. Yamaguchi, *Pattern of neutrino oscillations in supersymmetry with bilinear R-parity violation*, *Phys. Lett.* **B 476** (2000) 116 [hep-ph/9910320].

[6] M. Hirsch, M.A. Diaz, W. Porod, J.C. Romao and J.W.F. Valle, *Neutrino masses and mixings from supersymmetry with bilinear R-parity violation: a theory for solar and atmospheric neutrino oscillations*, hep-ph/0004115.

[7] O.C. W. Kong, *Neutrino oscillations and flavor structure of supersymmetry without R-parity*, *Mod. Phys. Lett.* **A 14** (1999) 903 [hep-ph/9808304].

JHEP09(2000)037

[8] A. Abada and M. Losada, *Constraints on a general 3-generation neutrino mass matrix from neutrino data: application to the MSSM with R-parity violation*, *Nucl. Phys.* **B 585** (2000) 45 [hep-ph/9908352].

[9] K. Cheung and O.C. W. Kong, *Zee neutrino mass model in SUSY framework*, *Phys. Rev.* **D 61** (2000) 113012 [hep-ph/9912238].

[10] Y. Grossman and H.E. Haber, *Neutrino masses and sneutrino mixing in R-parity violating supersymmetry*, hep-ph/9906310;
see also M. Hirsch, H.V. Klapdor-Kleingrothaus and S.G. Kovalenko, *B-L violating masses in softly broken supersymmetry*, *Phys. Lett.* **B 398** (1997) 311 [hep-ph/9701253].

[11] Super-Kamiokande collaboration, *Evidence for oscillation of atmospheric neutrinos*, *Phys. Rev. Lett.* **81** (1998) 1562 [hep-ex/9807003];
P. Lipari, *Atmospheric neutrinos: phenomenological summary and outlook*, hep-ph/9904443;
G.L. Fogli, E. Lisi, A. Marrone and G. Scioscia, Super-Kamiokande *atmospheric neutrino data, zenith distributions and three-flavor oscillations*, *Phys. Rev.* **D 59** (1999) 033001 [hep-ph/9808205].

[12] E.J. Chun and S.K. Kang, *One-loop corrected neutrino masses and mixing in supersymmetric standard model without R-parity*, *Phys. Rev.* **D 61** (2000) 075012 [hep-ph/9909429].

[13] M. Bisset, O.C. W. Kong, C. Macesanu and L.H. Orr, *A simple phenomenological parametrization of supersymmetry without R-parity*, *Phys. Lett.* **B 430** (1998) 274 [hep-ph/9804282];
Supersymmetry without R-parity: leptonic phenomenology, *Phys. Rev.* **D 62** (2000) 035001 [hep-ph/9811498].

[14] Y.Y. Keum and O.C. W. Kong, *R-parity violating contribution to neutron EDM at One-loop*, hep-ph/0004110.

[15] Y.-Y. Keum and O.C.W. Kong, IPAS-HEP-k006, manuscript in preparation.

[16] Y.-Y. Keum and O.C.W. Kong, work in progress.

[17] K. Cheung and O.C.W. Kong, IPAS-HEP-k007, manuscript in preparation.

[18] L.E. Ibáñez and G.G. Ross, *Discrete gauge symmetries and the origin of baryon and lepton number conservation in supersymmetric versions of the standard model*, *Nucl. Phys.* **B 368** (1992) 3.

[19] O.C.W. Kong, IPAS-HEP-k008, manuscript in preparation.

[20] S.Y. Choi, E.J. Chun, S.K. Kang and J.S. Lee, *Neutrino oscillations and R-parity violating collider signals*, *Phys. Rev.* **D 60** (1999) 075002 [hep-ph/9903465].

[21] See Y. Grossman and H.E. Haber, *(S)neutrino properties in R-parity violating supersymmetry, 1. CP-conserving phenomena*, *Phys. Rev.* **D 59** (1999) 093008 [hep-ph/9810536].

[22] A. Pilaftsis, *Higgs scalar-pseudoscalar mixing in the minimal supersymmetric standard model*, *Phys. Lett.* **B 435** (1998) 88 [hep-ph/9805373];
A. Pilaftsis and C.E.M. Wagner, *Higgs bosons in the minimal supersymmetric standard model with explicit CP-violation*, *Nucl. Phys.* **B 553** (1999) 3 [hep-ph/9902371].

[23] A. Zee, *A theory of lepton number violation, neutrino majorana mass and oscillation*, *Phys. Lett.* **B 93** (1980) 389.

[24] S. Davidson and M. Losada, *Neutrino masses in the R_p-violating MSSM*, *J. High Energy Phys.* **05** (2000) 021 [hep-ph/0005080].